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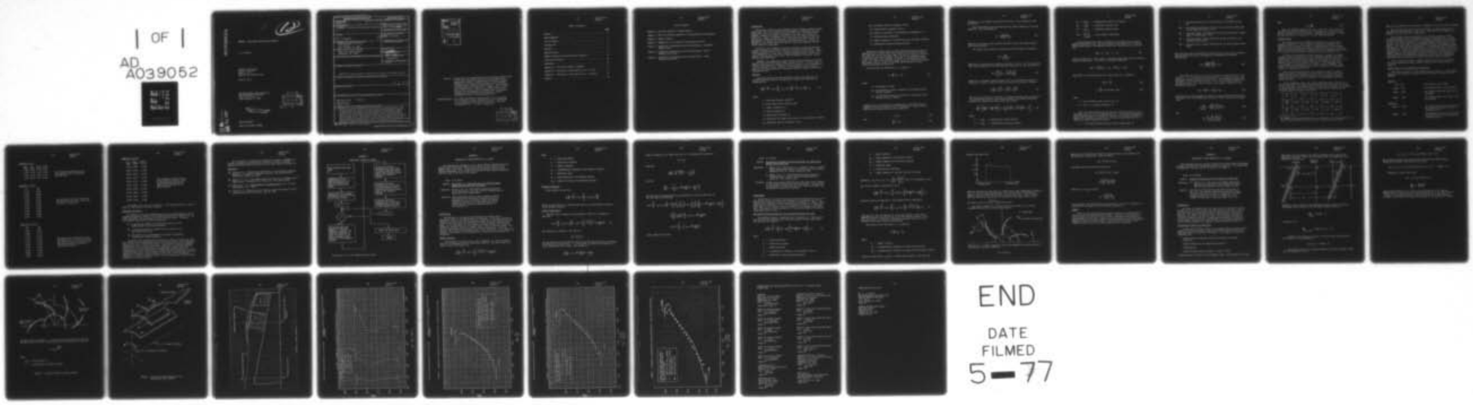

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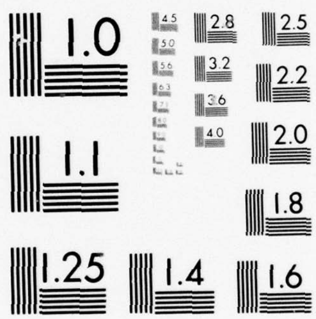

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PRESKEW: PRELIMINARY SKEW ANALYSIS PROGRAM

A. L. Treaster

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Presented in this memorandum are the theoretical modifications to the radial equilibrium equation and the computerized adaptations required to approximate the effects of skewed blading in the current working version of the streamline curvature method computer program (SCM-IV). The procedure is illustrated by the analysis of a pumpjet that is fitted with a skewed rotor. The results of this analysis which utilizes the data generated by PRESKEW show an improved agreement with experimental data relative to SCM-IV calculations prior to the skew modification.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Abstract: Presented in this memorandum are the theoretical modifications to the radial equilibrium equation and the computerized adaptations required to approximate the effects of skewed blading in the current working version of the streamline curvature method computer program (SCM-IV). The procedure is illustrated by the analysis of a pumpjet that is fitted with a skewed rotor. The results of this analysis which utilizes the data generated by PRESKEW show an improved agreement with experimental data relative to SCM-IV calculations prior to the skew modification.

Acknowledgments: The author expresses his appreciation to E. P. Bruce and M. W. McBride for their contributions to the development of the preliminary skew analysis procedure. This work was sponsored by T. E. Peirce, NAVSEA 0351.

Table of Contents

List of Figures

Introduction

The initial version of the streamline curvature method computer program (SCM-I, Ref. [a]) for the analysis of axisymmetric turbomachine flows was applicable only to flow regions outside of blade rows. To design skewed blading, a method was needed to approximate the flow effects within the blade rows. Bruce, in Appendix B, has shown that the radial equilibrium equation for application within blade rows developed by Smith in Ref. [b] was amenable to solution by streamline curvature techniques. McBride in Appendix C, has offered additional physical supporting arguments relevant to the effects of blade skew.

This paper will summarize the work of the three previously mentioned authors and present the approach used to successfully modify SCM-I. (The current, working version of the streamline curvature method program, SCM-IV, to be documented at a later date, includes approximations of shroud inlet losses, blade secondary flow losses, and the effects of preswirl, counter-rotation, and blade skew.)

PRESKEW is an independent computer program which develops the required data for the utilization of the "skew option" in SCM-IV. At this time, it should be emphasized that PRESKEW and SCM-IV represent a first approximation of the blade skew effects. However, the comparative results presented later in this paper demonstrate encouraging agreement between computed and experimental data.

Analysis

When heat addition and compressibility effects are neglected, the radial equilibrium equation for application within blade rows has the following form:

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{V_\theta^2}{r} + \cos\phi\,\frac{V_m^2}{R_m} - V_m \sin\phi\,\frac{\partial V_m}{\partial m} + \frac{V_z}{\lambda}\tan\phi\,\frac{D\lambda}{Dz} + F_r \qquad (1)$$

where

ρ = fluid mass density, slugs/ft^3

P = local fluid static pressure, psfa

r = radial coordinate, ft

z = axial coordinate, ft

m = meridional coordinate, ft

ϕ = meridional flow angle with respect to the centerline, radians

V_θ = peripheral velocity component, ft/sec

V_m = meridional velocity component, ft/sec

V_z = axial velocity component, ft/sec

R_m = radius of curvature of the meridional streamlines, ft

λ = effective area coefficient

F_r = radial component of the body force per unit mass, lbs/slug

D = differential change following particle.

Equation (1) differs from the radial equilibrium equation that is applied outside of blade rows by the presence of the last two terms on the right-hand side of the equal sign. The first of these two terms is a function of the blade blockage. While the F_r term depends, inpart, on the blade skew. With reference to Figure 1, the local blade skew angle, ξ, is defined as the angle formed in the R-θ radial plane between the tangent line at a point to the blade mean-thickness line and a radial line through the same point.

The effective area coefficient, λ, is given by

$$\lambda = \frac{N}{2\pi}(\theta_s - \theta_p) \tag{2}$$

where

N = the number of blades

θ_s = the peripheral angular coordinate of the blade suction surface, radians

θ_p = the peripheral angular coordinate of the adjacent blade pressure surface, radians.

Although Bruce has indicated the significance of the $D\lambda/Dz$ term in Appendix B, most ARL blades are thin and parallel at a given section. Thus, when the blade boundary layer is neglected

$$\lambda \cong 1.0 \tag{3}$$

and

$$\frac{D\lambda}{Dz} \simeq 0.0 \quad . \tag{4}$$

Therefore, for the PRESKEW approximation the first of the additional terms is omitted.

Bruce and McBride have both shown that the radial component of the body force, F_r, can be expressed as

$$F_r = \frac{\Delta P \tan \xi}{\rho r(\theta_s - \theta_p)} \quad , \tag{5}$$

where ΔP is the local static pressure difference across the blade pressure and suction surfaces.

If a local static pressure coefficient across the blade is defined as

$$C_p' = \frac{\Delta P}{1/2 \rho V_\infty^{\ 2}} \quad ,$$

where V_∞ is a free stream or reference velocity in ft/sec, then the dimensionless radial component of the body force per unit mass, F_Y, can be written as

$$F_Y = \left(\frac{R_o}{V_\infty^{\ 2}} \right) F_r = \frac{N \tan \xi \, C_p'}{4\pi\lambda \ (r/R_o)} \tag{6}$$

where R_o is a reference radial distance such as the maximum body radius or propeller tip radius. When the radial derivative of Bernoulli's Equation

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = \frac{1}{\rho}\frac{\partial P_t}{\partial r} - \frac{1}{2}\frac{\partial V_m^2}{\partial r} - \frac{1}{2}\frac{\partial V_\theta^2}{\partial r} \quad , \tag{7}$$

where P_t is the fluid total pressure, is combined with Equations (1) and (6) in a dimensionless manner, the "skewed" version of the radial equilibrium equation for computations within blade rows results:

$$\frac{\partial CM}{\partial Y} + \left[\frac{Cos\phi}{RC} - \frac{Sin\phi}{CM}\frac{\partial CM}{M} \right] CM = \frac{1}{CM}\left[\frac{1}{2}\frac{\partial CPT}{\partial Y} - \frac{CU}{Y}\frac{\partial (Y \cdot CU)}{\partial Y} - F_Y \right] \quad , \tag{8}$$

where

$Y \quad = \quad r/R_o \quad = \quad$ dimensionless radial distance

$M \quad = \quad m/R_o \quad = \quad$ dimensionless meridional distance

$RC = R_m/R_o$ = dimensionless radius of curvature

$CM = V_m/V_\infty$ = meridional velocity ratio

$CU = V_\theta/V_\infty$ = peripheral velocity ratio

$CPT = \dfrac{P_t - P}{1/2\rho V_\infty^2}$ = total pressure coefficient

To apply Equation (8) within the blade row the changes in the angular momentum coefficient, ΔM_θ , and total pressure coefficient, ΔCPT , from the blade leading edge to the computing station must be specified as a function of Y.

$$\Delta M_\theta = Y_2 \cdot CU_2 - Y_1 \cdot CU_1 \quad , \tag{9}$$

where the subscripts 1 and 2 refer to the blade leading edge and the computing station, respectively. From Euler's turbomachine equation

$$\Delta CPT = 2\left((\overline{U}/V_\infty)_2 \cdot CU_2 - (\overline{U}/V_\infty)_1 \cdot CU_1\right) \quad , \tag{10}$$

where $\overline{U}/V_\infty$ is the dimensionless rotor wheel speed and is defined by

$$\overline{U}/V_\infty = \frac{\pi Y}{J} \quad . \tag{11}$$

$$J = \frac{V_\infty}{nD} = \text{the advance ratio} \quad , \tag{12}$$

where

n = the rotational speed of the rotor, rps

$D = 2R_o$ = a reference diameter, ft.

When J has been specified ΔM_θ and ΔCPT are functions only of CU for a particular value of Y at the computing station. For blades designed by the mean streamline method of Wislicenus, Reference [c], $\Delta V_\theta/V\theta_t$ and $\Delta P/\overline{\Delta P}$ are empirically specified as a function of the axial blade length ratio x/ℓ, where at any blade cylindrical section

x = the axial distance from the blade's leading edge, ft

ℓ = the axial projection of the blade chord at the given section, ft

$\Delta V\theta$ = the change in peripheral velocity from the blade's leading edge to the computing station, ft/sec

$V\theta_t$ = the total change in peripheral velocity from the blade's leading edge to trailing edge, ft/sec

ΔP = the local static pressure difference across the blade at the computing station, psfa

$\overline{\Delta P}$ = the mean static pressure difference for the entire blade section, psfa.

Once these data have been specified, Equations (9) and (10) can be evaluated from the known CU_1 versus Y distribution by employing the following relationship:

$$CU_2 = \left(\frac{\Delta V\theta}{V\theta_t}\right)\left(\frac{V\theta_t}{V_\infty}\right) + CU_1 \quad . \tag{13}$$

The $\Delta P/\overline{\Delta P}$ distribution is used to calculate the radial body force parameter, F_Y. If ξ is specified as a function of Y, F_Y, Equation (6), for any cylindrical section becomes a function of x/ℓ. For the fluid element shown in Figure 2 which has dimensions t by ℓ by dr, the peripheral component of the pressure force, dF_θ , must be equal to the change of momentum in the peripheral direction:

$$dF_\theta = \overline{\Delta P}\ell dr = \rho\overline{V_a}tdr\ \Delta V\theta_t \quad , \tag{14}$$

where $\overline{V_a}$ is an average meridional velocity for the particular blade section. If Equation (14) is rearranged and divided by $1/2\rho V_\infty{}^2$, C_p' can be calculated from

$$\frac{\overline{\Delta P}}{1/2\rho V_\infty{}^2} = 2\left(\frac{\overline{V_a}}{V_\infty}\right)\left(\frac{\Delta V\theta_t}{V_\infty}\right)\left(\frac{t}{\ell}\right) \tag{15}$$

and

$$C_p' = \left(\frac{\overline{\Delta P}}{1/2\rho V_\infty{}^2}\right)\left(\frac{\Delta P}{\overline{\Delta P}}\right) \quad , \tag{16}$$

where

$$\frac{t}{\ell} = \frac{2\pi r}{N\ell} \quad . \tag{17}$$

Thus, by combining Equations (6), (9), (10), (11), (12), (13), (15), and (16), the necessary parameters for the utilization of Equation (8) within the blade row can be calculated. PRESKEW was developed to compute F_Y, ΔM_θ, and ΔCPT as a function of Y at the computing station.

Computerization

The logical operation of PRESKEW is documented by the functional diagram of Appendix A. The following discussion will summarize the operation of the program and explain the approximations and procedures used to calculate F_Y, ΔM_θ, and ΔCPT as a function of Y for use in SCM-IV. Most of the data specification, interpolation, and integration is performed by the spline curve procedures documented by Davis in Reference [d].

To avoid the convergence problems inherent to streamline curvature techniques when using high aspect ratio computing stations, one* axial station within the blade row was chosen for the radial specification of the skew effects. The blade planform shape is represented by four intersecting straight lines as depicted in Figure 3. The blade skew angle, ξ, is assumed to be constant for the current version of PRESKEW. This assumes that the blade mean-thickness line is described by a logarithmic spiral as illustrated in Figure 1.

Once the input data have been read, computation begins by approximating the $(\overline{V}_a/V_\infty)$ distribution as a function of Y along a radial line midway between the two computing stations. This calculation is based on the continuity principle and integration of the reference meridional velocity profile.

As the computing station, the radial computation points (values of Y) and the projected local blade section length are computed at each 10 percent of the blade span. The required values of $(\Delta V\theta/V\theta_t)$ and $(\Delta P/\overline{\Delta P})$ are calculated at the appropriate values of x/ℓ from the spline curves representing the following typical ARL section properties:

$\frac{x}{\ell}$	0.0	0.2	0.4	0.8	1.0
$\frac{\Delta V\theta}{V\theta_t}$	0.000	0.180	0.427	0.890	1.000
$\frac{\Delta P}{\overline{\Delta P}}$	0.500	0.978	0.171	1.127	0.000

*In PRESKEW, two computing stations are used; however, only the parameters from one of these stations are used to specify the skew effects in SCM-IV.

These section properties were supplied to PRESKEW in the form of data statements.

Once the values of CU_2 and C_p' are calculated from data obtained from the spline curves (Equations 13 and 15), F_Y, $\Delta M\theta$, and ΔCPT are computed from Equations (6), (9), and (10), respectively.

The F_Y versus Y data are supplied to the mainline program of SCM-IV, as data statements and are used in the evaluation of Equation (8). The ΔM_θ and ΔCPT data versus Y are supplied to the ROTO4 subroutine of SCM-IV, again, in the form of data statements. If a second skewed, rotating blade row is present in the flow field, a separate PRESKEW analysis is performed for this blading. The resulting output parameters at the designated computing station are supplied to the mainline program and to the STTR4 subroutine of SCM-IV via data statements. Thus, the radial distribution of the blade skew effects are specified at one computing station within the blade row. The effects of the skewed blading on the neighboring stations are affected by the shifting of the spline curves representing the streamline pattern and the accompanying redistribution of the meridional velocity and static pressure profiles.

In the following section, a sample calculation based on a recent wind tunnel test program employing a pumpjet propulsor with a skewed rotor is presented.

Sample Calculation and Experimental Comparison

The subject propulsor is shown schematically in Figure. 3. The required input data are presented in tabular form to illustrate the input format to PRESKEW.

Card No. 1

XN = 13.0	the number of rotor blades
XLAM = 1.00	the blockage effects were neglected
SKEW = 0.694	the constant blade skew angle (39.7°) in radians
ADRAT = 2.092	the average advance ratio used in the test program

Card No. 2

XXX(1) = 1.389	the dimensionless axial distance from the reference station to the first computing station
XXX(2) = 1.495	the dimensionless axial distance from the reference station to the second computing station

Cards No. 3 to 6

X(I)	Y(I)	XX(I)	YY(I)
1.204	0.361	1.279	0.710
1.279	0.710	1.654	0.639
1.575	0.282	1.654	0.639
1.204	0.361	1.575	0.282

The end-point coordinates of the four straight lines representing the blade row.

Cards No. 7 to 16

Y_1	CU_1
0.361	0.000
0.400	0.000
0.440	0.000
0.480	0.000
0.520	0.000
0.550	0.000
0.570	0.000
0.590	0.000
0.610	0.000
0.639	0.000

The peripheral velocity distribution at the rotor-inlet plane. There was no preswirl in the flow.

Cards No. 17 to 26

Y_E	CU_E
0.282	0.553
0.300	0.523
0.350	0.467
0.400	0.440
0.425	0.420
0.450	0.400
0.500	0.366
0.550	0.348
0.600	0.350
0.639	0.402

The experimental peripheral velocity distribution at the rotor-exit plane. For initial blade specification, this distribution would be dictated by the design conditions.

Cards No. 27 to 36

Y_{REF}	CM_{REF}	ANGL(I)
0.611	0.500	- 0.209
0.625	0.590	- 0.209
0.650	0.665	- 0.209
0.675	0.707	- 0.209
0.700	0.745	- 0.209
0.725	0.777	- 0.209
0.750	0.805	- 0.209
0.765	0.822	- 0.209
0.785	0.842	- 0.209
0.810	0.865	- 0.209

The meridional velocity profile at the reference station. This is the same reference profile that will also be used in the SCM-IV analysis.

The PRESKEW output data for the XXX(1) = 1.389 computing station, shown in Figure 4, were used in the SCM-IV analysis.

Discussion of Results

The results of using the PRESKEW output data in a SCM-IV analysis of the pumpjet represented by Figure 3 are graphically presented in Figures 4, 5, and 6 for measurements and calculations at the rotor exit plane. In each of these figures three types of data are presented:

(1) a theoretical analysis by SCM-IV with shroud inlet and blade secondary flow losses included,

(2) a theoretical analysis by SCM-IV with the losses of (1) plus skew effects, and

(3) the results of an experimental wind tunnel investigation utilizing the subject propulsor.

In general, the calculated data including the effects of blade skew agree more closely with the experimental data than does the SCM-IV results employing only the inlet and secondary flow losses. As predicted by Bruce and McBride (Appendixes B and C). the total pressure coefficient is relatively unaltered by the effect of blade skew, whereas, the meridional velocity exhibits a retardation in the tip region with an accompanying acceleration near the root; correspondingly, there is an increase in static pressure coefficient in the tip region and a decrease in static pressure at the root. These changes are beneficial in reducing cavitation in the rotor tip region and in delaying flow separation at the root section.

As is required, a variable skew option will be added to PRESKEW; and as more is understood concerning the streamline curvature convergence problems additional computing stations will be added within the blade row.

References

[a] Treaster, A. L. "Computerized Application of the Streamline Curvature Method to the Indirect Turbomachine Problem," ARL Unclassified TM 514,2491-16, October 31, 1969.

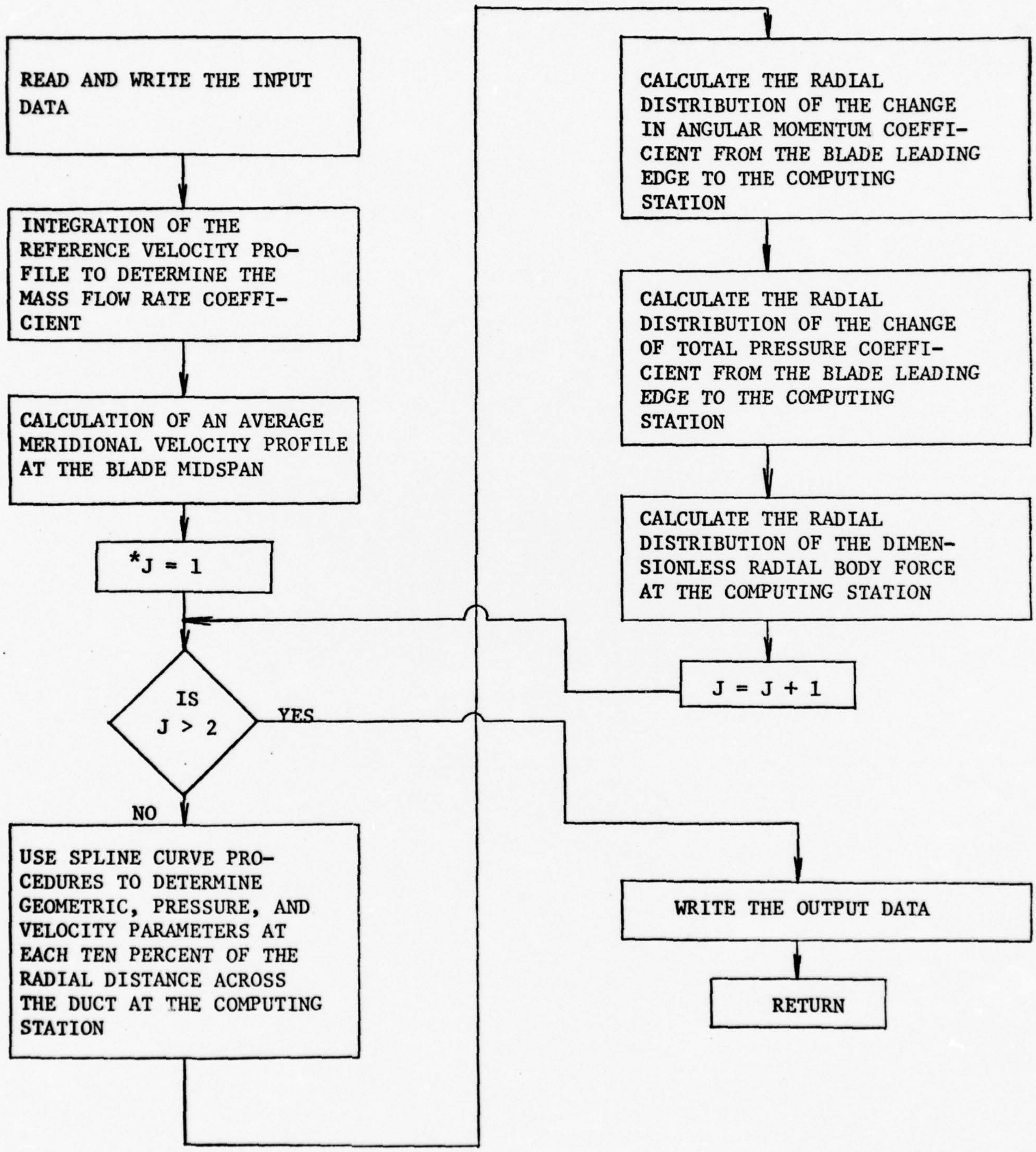
[b] Smith, L. H., Jr., "The Radial Equilibrium Equation of Turbomachinery," ASME Journal of Engineering for Power, pp. 1-12, January 1966.

[c] Wislicenus, G. F., Fluid Mechanics of Turbomachinery, Vol. II, Dover Publications, Inc., 1965.

[d] Davis, R. F., "Spline Curve Fit Functions, Their Derivatives and Use," ARL Unclassified TM 512-3531-02, July 23, 1968.

APPENDIX A

FUNCTIONAL DIAGRAM OF PRESKEW



*As used here, J is the computing station index.

9 March 1977
ALT:jep

APPENDIX B

DISCUSSION OF SKEW EFFECTS BY E. P. BRUCE

The following two documents by E. P. Bruce contain a discussion of the general radial equilibrium equation and explain how the equation can be applied within blade rows. These papers were originally written as ARL Internal Memoranda; and no attempt has been made to match their nomenclature with that used in the body of this report.

From: E. P. Bruce

Subject: Proof That A. L. Treaster's and L. H. Smith's Radial Equilibrium Equations are Equivalent

Reference: Smith, L. H., Jr., "The Radial Equilibrium Equation of Turbomachinery," A.S.M.E. Journal of Engineering for Power, pp. 1-12, January 1966.

Abstract: This memorandum presents a short algebraic exercise which proves that the radial equilibrium equation developed for use in the Water Tunnel SCM computer program is identical to a simplified form of the general radial equilibrium equation derived by L. H. Smith, Jr.

Introduction

In Reference 1, the general three-dimensional unsteady radial equilibrium equation of turbomachinery is developed. A rotating, cylindrical coordinate system is used and the resulting equation is then compared with the equation that would have been obtained if a fixed coordinate system had been used. In the development of the Streamline Curvature Method (SCM) computer program at the Water Tunnel, Al Treaster used a fixed coordinate system. Thus the two independently derived equations should be equivalent. Since the equations are presented in different forms, their equivalence is not immediately obvious. The purpose of this memorandum is to show that the equations are equivalent.

Smith's Equation

The equation of interest is Eq. (31) of Reference 1. This is an exact form of the axisymmetric radial equilibrium equation. With Mach Numbers set equal to zero, this equation is

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{C_u^2}{r} + \sec^3\phi\,\frac{C_z^2}{r_m} + \frac{C_z^2 \tan\phi}{r}\,\frac{\partial(r\tan\phi)}{\partial r}$$

where

ρ = fluid mass density

p = fluid static pressure

r = radial coordinate

C_u = circumferential component of the absolute velocity

ϕ = meridional angle

C_z = axial component of the absolute velocity

r_m = radius of curvature of meridional pathline

Treaster's Equation

This equation has the form

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{C_u^2}{r} + \cos\phi\,\frac{C_m^2}{r_m} - C_m \sin\phi\,\frac{\partial C_m}{\partial m}$$

where all quantities are as defined above and C_m is the meridional component of the absolute velocity.

Proof of Equivalence

The C_u^2/r term is common to both equations. Hence it is necessary to show that

$$\cos\phi\,\frac{C_m^2}{r_m} - C_m \sin\phi\,\frac{\partial C_m}{\partial m} = \sec^3\phi\,\frac{C_z^2}{r_m} + \frac{C_z^2 \tan\phi}{r}\,\frac{(r\tan\phi)}{\partial r} \qquad (1)$$

Two relations are required. The first is

$$C_m = C_z/\cos\phi$$

and the second can be obtained from Smith's Eq. (81) which relates the variation of meridional streamtube area, A , to variations in meridional streamtube, ϕ , for an axisymmetric flow field. This equation is

$$\frac{1}{A}\frac{\partial A}{\partial m} = \cos\phi\,\frac{\partial(r\tan\phi)}{r\partial r} + \frac{\tan\phi}{r_m}$$

Along a streamtube, the volume flow rate, Q , is constant and is given by

$$Q = A\, C_m$$

From this

$$\frac{1}{A}\frac{\partial A}{\partial m} = \frac{C_m}{Q}\frac{\partial (Q/C_m)}{\partial m} = -\frac{1}{C_m}\frac{\partial C_m}{\partial m}$$

and then

$$\frac{\partial C_m}{\partial m} = -C_m\left[\cos\phi\,\frac{\partial(r\tan\phi)}{r\partial r} + \frac{\tan\phi}{r_m}\right]$$

Substituting the expressions for C_m and $\partial C_m/\partial m$ into the left hand side of Eq. (1) yields the following:

$$\cos\phi\,\frac{C_m^2}{r_m} - C_m\sin\phi\,\frac{\partial C_m}{\partial m} = \frac{\cos\phi}{r_m}\left(\frac{C_z^2}{\cos^2\phi}\right) + \sin\phi\left(\frac{C_z^2}{\cos^2\phi}\right)\left[\cos\phi\,\frac{\partial(r\tan\phi)}{r\partial r} + \frac{\tan\phi}{r_m}\right]$$

$$= \frac{C_z^2}{r_m}\left(\frac{1+\tan^2\phi}{\cos\phi}\right) + C_z^2\tan\phi\,\frac{\partial(r\tan\phi)}{r\partial r}$$

$$= \sec^3\phi\,\frac{C_z^2}{r_m} + C_z^2\tan\phi\,\frac{\partial(r\tan\phi)}{r\partial r}$$

which completes the proof.

9 March 1977
ALT:jep

From: E. P. Bruce

Subject: The Form of the Radial Equilibrium Equation for Application Inside Skewed Blade Rows

References: 1) Bruce, E. P., "Proof That A. L. Treaster's and L. H. Smith's Radial Equilibrium Equations are Equivalent," ORL Internal Memorandum File No. 72-207, October 12, 1972.

2) Smith, L. H., Jr., "The Radial Equilibrium Equation of Turbomachinery," A.S.M.E. Journal of Engineering for Power, pp. 1-12, January 1966.

Abstract: A form of the radial equilibrium equation which can be applied in flow regions inside blade rows is presented. The new terms of the equation are discussed and their relation to known flow and geometric properties is defined.

Introduction

In reference 1, the radial equilibrium equation developed for use in the SCM computer program is shown to be identical to a simplified form of the general radial equilibrium equation developed in Reference 2. Both of these equations are applicable to flow regions outside of blade rows. Additional terms appear in the equation if it is applied to flow regions inside blade rows. The purpose of this memorandum is to present and discuss the equation containing these new terms.

The Radial Equilibrium Equation for Applications Within Blade Rows

The pertinent equation is Equation (51) of Reference 2. This equation has the following form when terms involving products of Mach numbers and heat addition are neglected

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{C_u^2}{r} - \frac{D^2 r}{Dz^2} W_z^2 + W_r W_z \left[\frac{\partial (r \tan \phi)}{r \partial r} + \frac{1}{\lambda}\frac{D\lambda}{Dz} \right] + F_r \qquad (1)$$

where

ρ = fluid mass density

p = fluid static pressure

r = radial coordinate

C_u = circumferential component of the absolute velocity

D = differential change following particle

z = axial coordinate

W_z = axial component of the relative velocity

W_r = radial component of the relative velocity

ϕ = meridional angle

λ = effective area coefficient

F_r = radial component of the body force per unit mass

Setting $W_r = W_z \tan\phi$, $W_z = C_z$, $\dfrac{D^2 r}{Dz^2} = \dfrac{W_m^2 \sec\phi}{W_z^2\, r_m}$ (Eq. 37 of Reference 2) and

$W_m = C_z \sec\phi$ permits rewriting Eq. (1) as

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{C_u^2}{r} + \sec^3\phi\,\frac{C_z^2}{r_m} + C_z^2 \tan\phi\left[\frac{\partial(r\tan\phi)}{r\partial r} + \frac{1}{\lambda}\frac{D\lambda}{Dz}\right] + F_r$$

Using the results of Reference 1, this equation may be expressed as

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{C_u^2}{r} + \cos\phi\,\frac{C_m^2}{r_m} - C_m \sin\phi\,\frac{\partial C_m}{\partial m} + \frac{C_z^2 \tan\phi}{\lambda}\,\frac{D\lambda}{Dz} + F_r \qquad (2)$$

This equation, with the omission of the last two terms, is the radial equilibrium equation presently used in the SCM computer program. In the last two terms, λ , $D\lambda/Dz$ and F_r represent quantities that are not now included or derivable within the SCM program.

The effective area coefficient, λ , is given by

$$\lambda = \frac{N}{2\pi}(\theta_s - \theta_p)$$

where

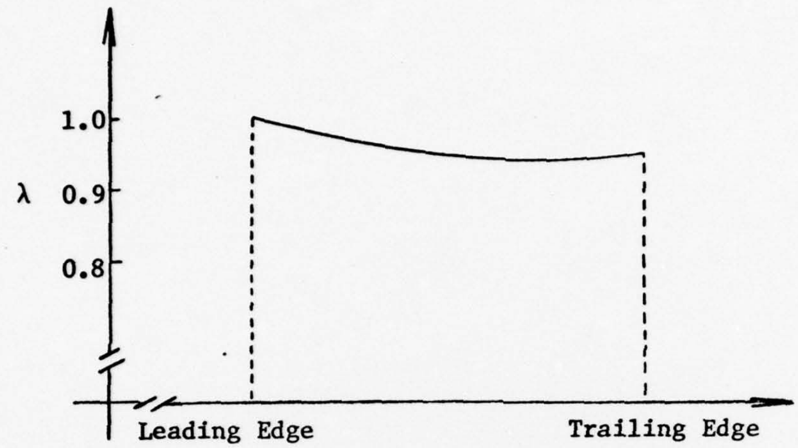
N = number of blades

θ_s = circumferential coordinate of blade suction surface

θ_p = circumferential coordinate of adjacent blade pressure surface

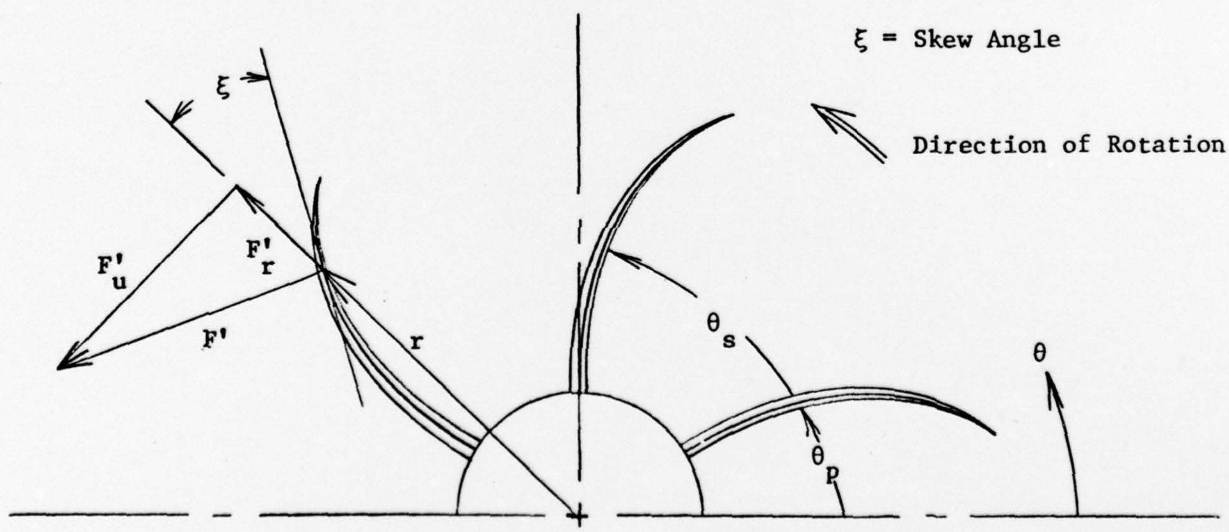
Thus at a fixed radius, a plot of λ versus axial distance z will have the

general form shown below



where an allowance has been made for blade boundary layer displacement thickness, i.e. $\lambda \neq 1$ at the blade trailing edge. This sketch shows the importance of taking a number of computing stations within the blade row. If, for example, only one station near the center of the blade row was taken, $D\lambda/Dz \approx 0$ and the effect of the term $\frac{C_z^2 \tan\phi}{\lambda} \frac{D\lambda}{Dz}$ would be lost.

The radial component of the body force per unit mass, F_r, is easily evaluated with the aid of the following sketch:



The force, F', on small element of area having dimensions ds along the blade surface and dz axially is given by

$$F' = \Delta p \; ds \; dz$$

where Δp is the difference between the static pressure on the pressure and suction faces, respectively. From the sketch,

$$F_r' = \Delta p \; ds \; dz \; \sin \xi_m$$

The radial force per unit mass is given by

$$F_r = F_r'/\rho \; dr \; r(\theta_s - \theta_p)dz$$

$$= \frac{\Delta p \sin \xi_m}{\rho r(\theta_s - \theta_p)} \left(\frac{ds}{dr}\right)$$

Now $dr/ds = \cos \xi_m$ so we have

$$F_r = \frac{\Delta p \tan \xi_m}{\rho r(\theta_s - \theta_p)}$$

In our designs, the quantities Δp, $\tan \xi_m$ and $(\theta_S - \theta_P)$ will, in general, be functions of the radius r and of the axial coordinate z.

Summary

A form of the radial equilibrium equation which can be applied within blade rcws has been presented and discussed. Inclusion of this equation in our SCM computer program will require that a preliminary blade design be completed which can be used to specify initial estimates of flow and geometric properties within the blade passage. The importance of taking a number of computing stations within the blade passage has also been noted.

9 March 1977
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APPENDIX C

DISCUSSION OF SKEW EFFECTS BY M. W. McBRIDE

This document which was initially presented as an ARL Internal Memorandum presents a discussion of the physical aspects of blade skew. The nomenclature is independent from that used in the body of this report.

From: M. W. McBride

Subject: Geometric Interpretation of the Effects of Blade Skew

Reference: (1) Bruce, E. P., "The Form of the Radial Equilibrium Equation for Application Inside Skewed Blade Rows," ORL Internal Memorandum 72-211, October 17, 1972

Abstract: The effect of blade skew has been shown previously in Ref. (1). This report intensifies the physical concept of skew effects and arrives at a relation for the radial pressure gradient due to skew in a blade row. This relation is identical to that determined in Ref. (1).

Introduction

The effect of backward skew in a blade row is to increase the static pressure at the blade tip and decrease the pressure at the root. This in turn decreases the tip meridional velocity and increases the velocity at the root. The radial equilibrium equation derived in Ref. (1) clearly indicates this effect, but does not give a clear physical indication of the actual mechanism involved. A simple geometric analysis is presented in this report which allows an understanding of skew while arriving at a radial pressure gradient term identical to that determined in Ref. (1).

The Geometric Analysis of Blade Skew

A radial plane through a blade row is characterized by N evenly spaced blades with arbitrary spanwise loading and thickness distributions and some skew angle as a function of the radius. At a radius, R , the blade properties are:

a blade pressure difference between the pressure and suction surface ΔP;

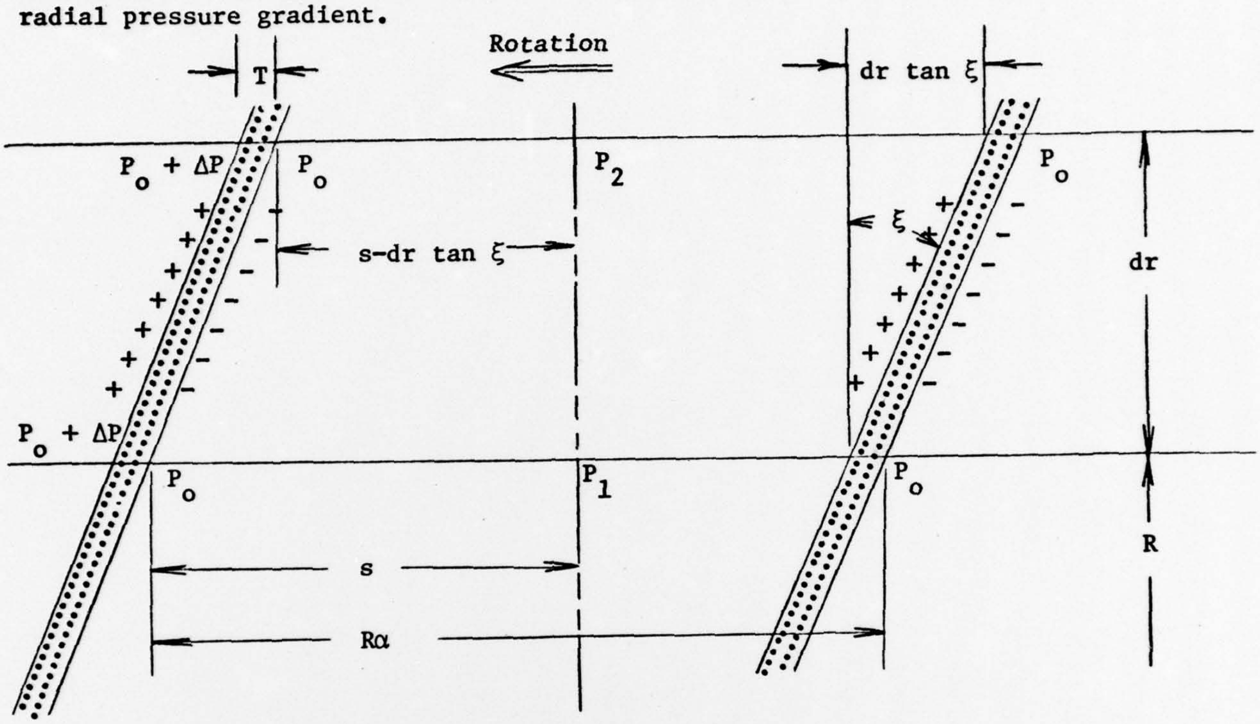
a blade thickness in the tangential direction T;

a skew angle ξ;

a blade-to-blade angular spacing, α , equal to $2\pi/N$.

These quantities are shown in the following figure. This analysis is for the

skew effect only and neglects the pressure gradient due to rotation and meridional curvature. These terms are simply added to determine the full radial pressure gradient.



Assuming a linear variation, the tangential pressure gradient at radius R from blade suction surface to the next blade pressure surface is

$$\frac{dP}{ds}_{(R)} = \Delta P/(R\alpha - T) \;.$$

At radius R + dr

$$\frac{dP}{ds}_{(R + dr)} = \Delta P/[(R + dr)\,\alpha - T] \;.$$

A value for the pressure a distance s from the blade suction surface at a radius R, can be determined;

$$P_1 = P_0 + s \cdot \Delta P/(R\alpha - T) \;.$$

The pressure along a line eminating radially from that previously found, but at a distance R + dr is,

$$P_2 = P_0 + (s - dr \tan \xi)\, \Delta P[(R + dr)\, \alpha - T] \,.$$

The difference between these two pressures will give the radial pressure change that the blades are exerting on the fluid;

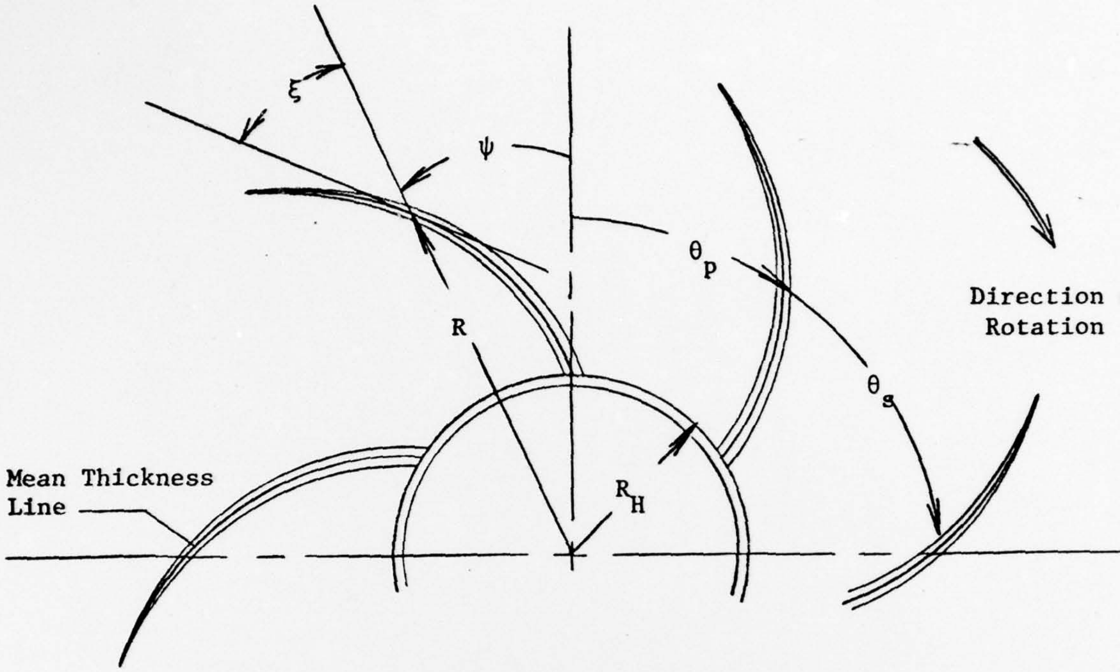
$$dP = P_2 - P_1 = (s - dr \tan \xi)\, \Delta P/[(R + dr)\, \alpha - T] - \Delta P/(R\alpha - T) \,.$$

Dropping the second order terms:

$$dP = - dr \tan \xi \, \Delta P/(R\alpha - T)$$

$$\frac{dP}{dr_B} = - \frac{\Delta P \tan \xi}{R\alpha - T} \,.$$

The pressure gradient in the fluid is then the negative of the quantity dP/dr_B, which is identical to the term F_R defined in Ref. (1). This gradient is seen to be uniform everywhere between the blades and is unaffected by crossing a blade surface, although the magnitude of the pressure will change.



For blades whose skew angle, ξ , is invariant with radial distance the mean thickness line must follow a logarithmic spiral whose equation is given by

$$R = R_H \, e^{\frac{\psi}{\tan \xi}}$$

where

R_H = the hub radius, ft

ψ = the peripheral coordinate, radians

FIGURE 1. The Basic Geometry of Skewed Blading

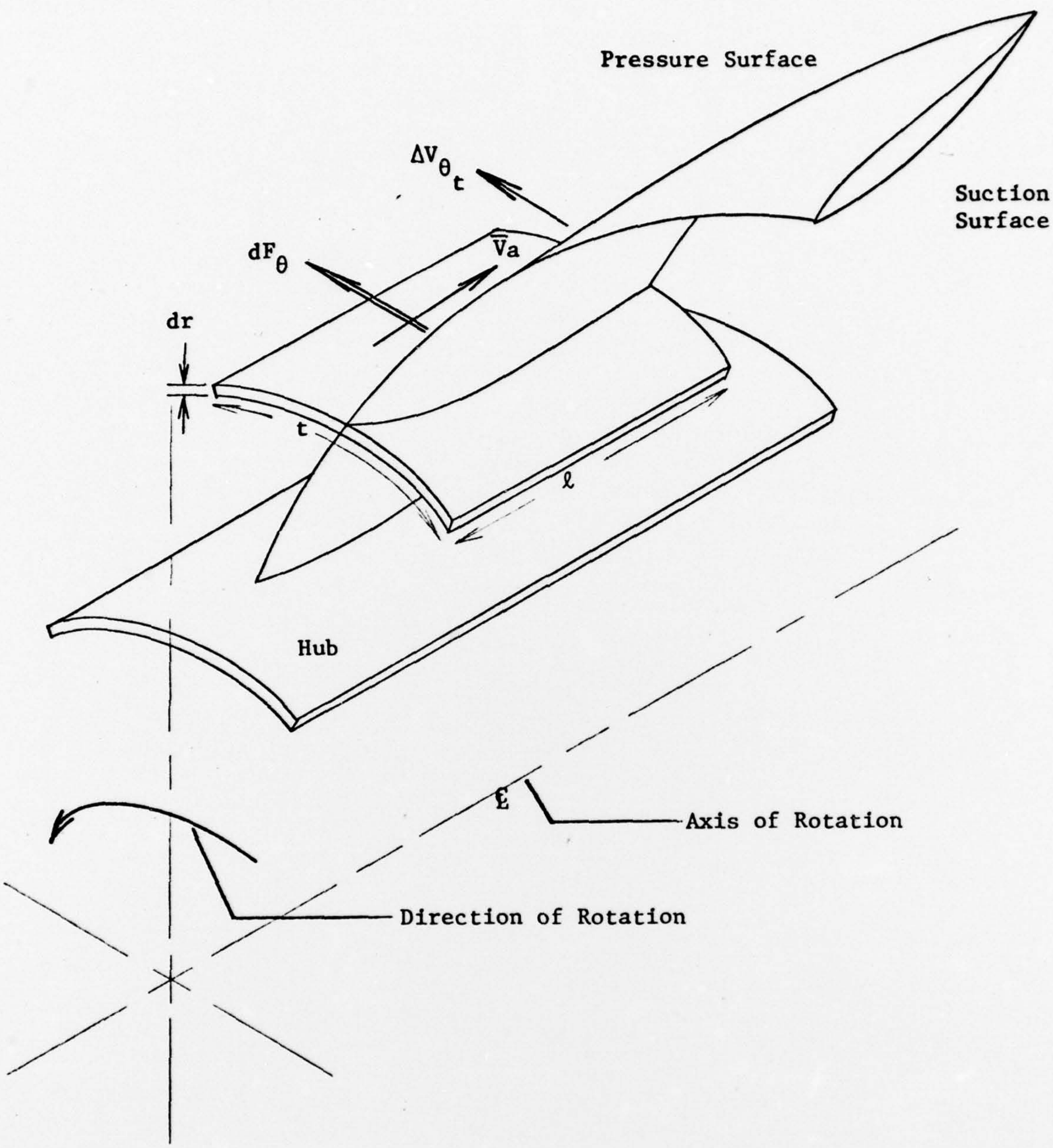


FIGURE 2. Force and Velocity Diagram for the Differential Fluid Element

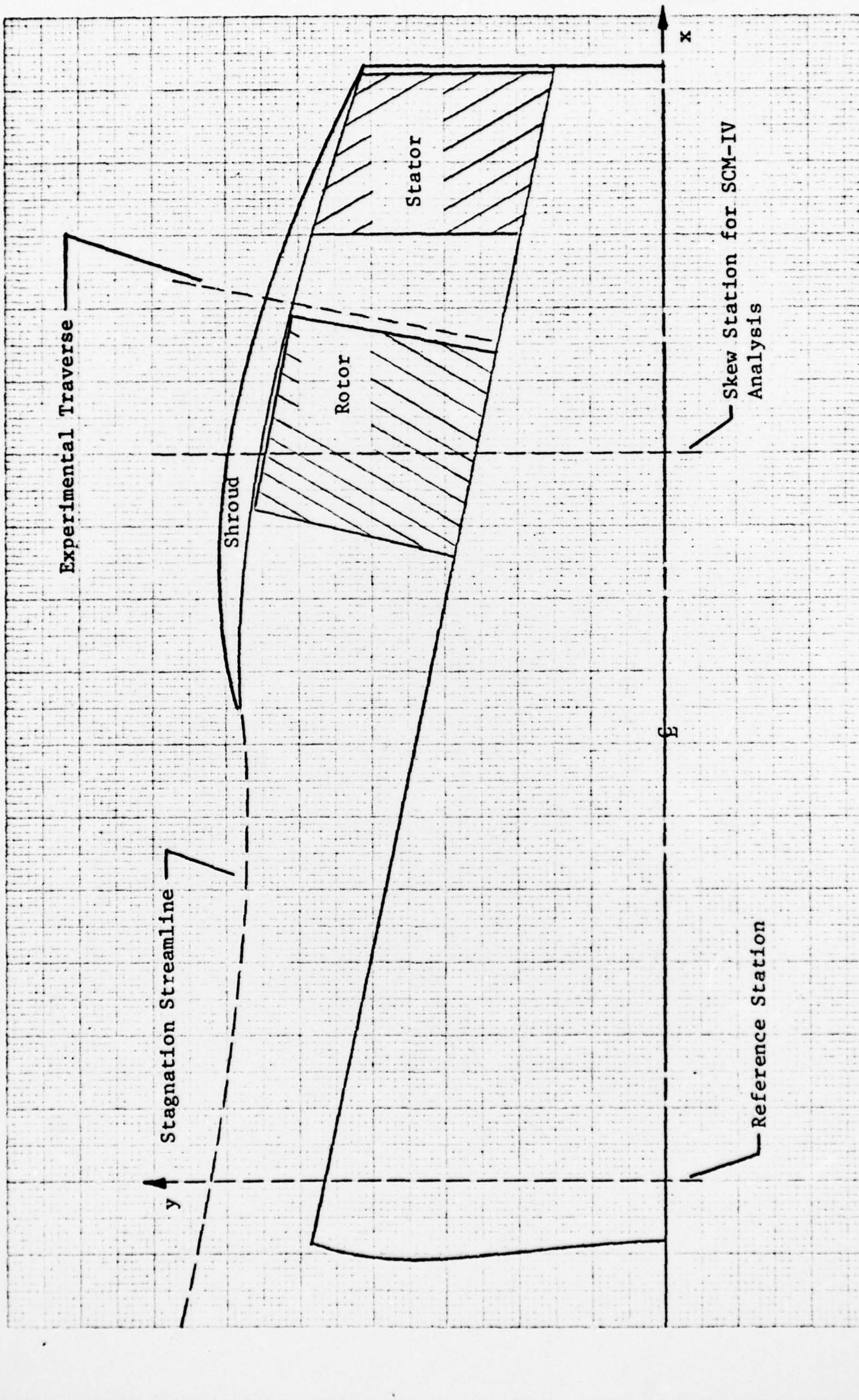


FIGURE 3. Pumpjet Geometry

9 Marcy 1977
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FIGURE 4. PRESKEW Data at the XXX(1)=1.389 Computing Station

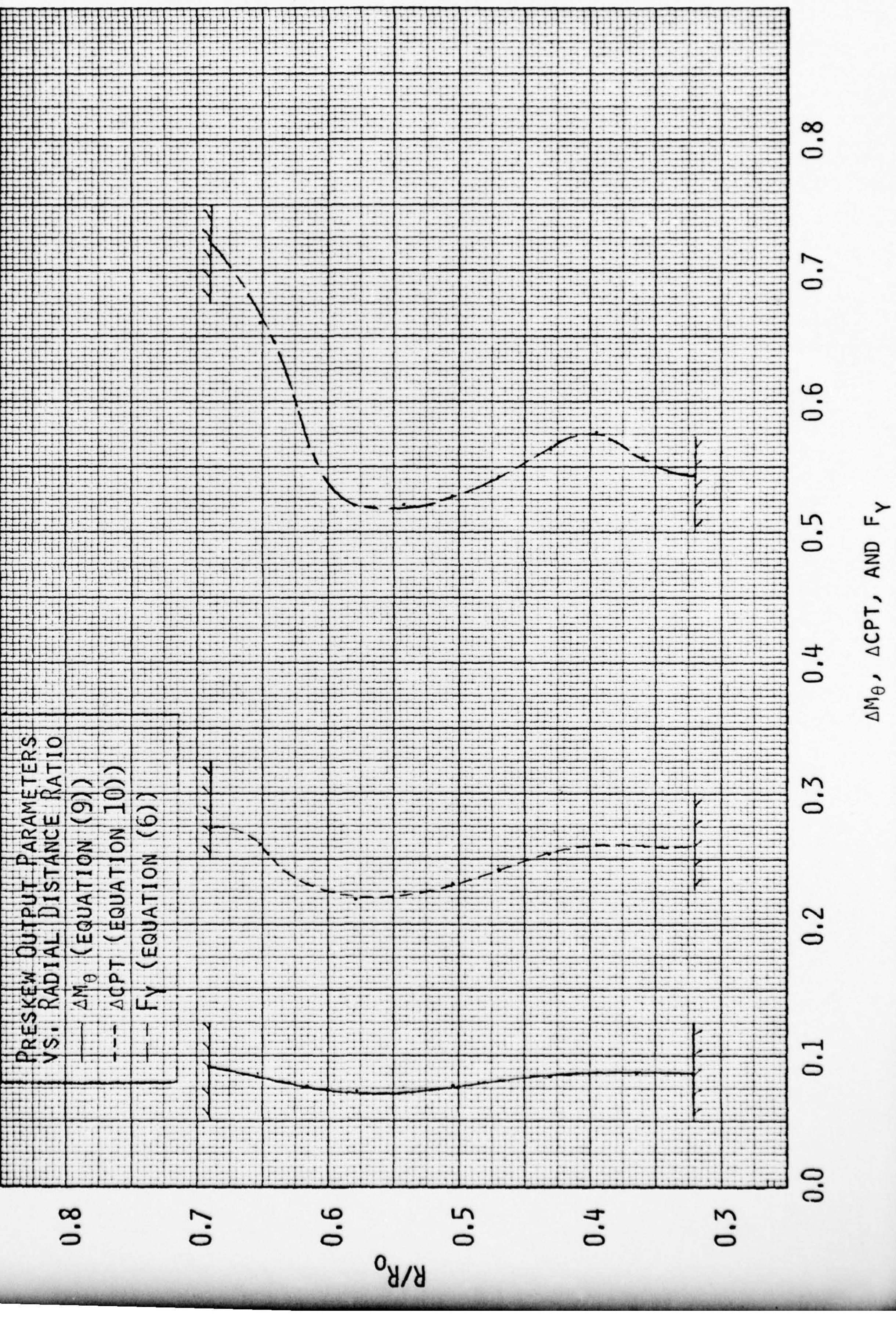

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FIGURE 5. Comparison of Experimental and Calculated Data: Meridional Velocity Ratio

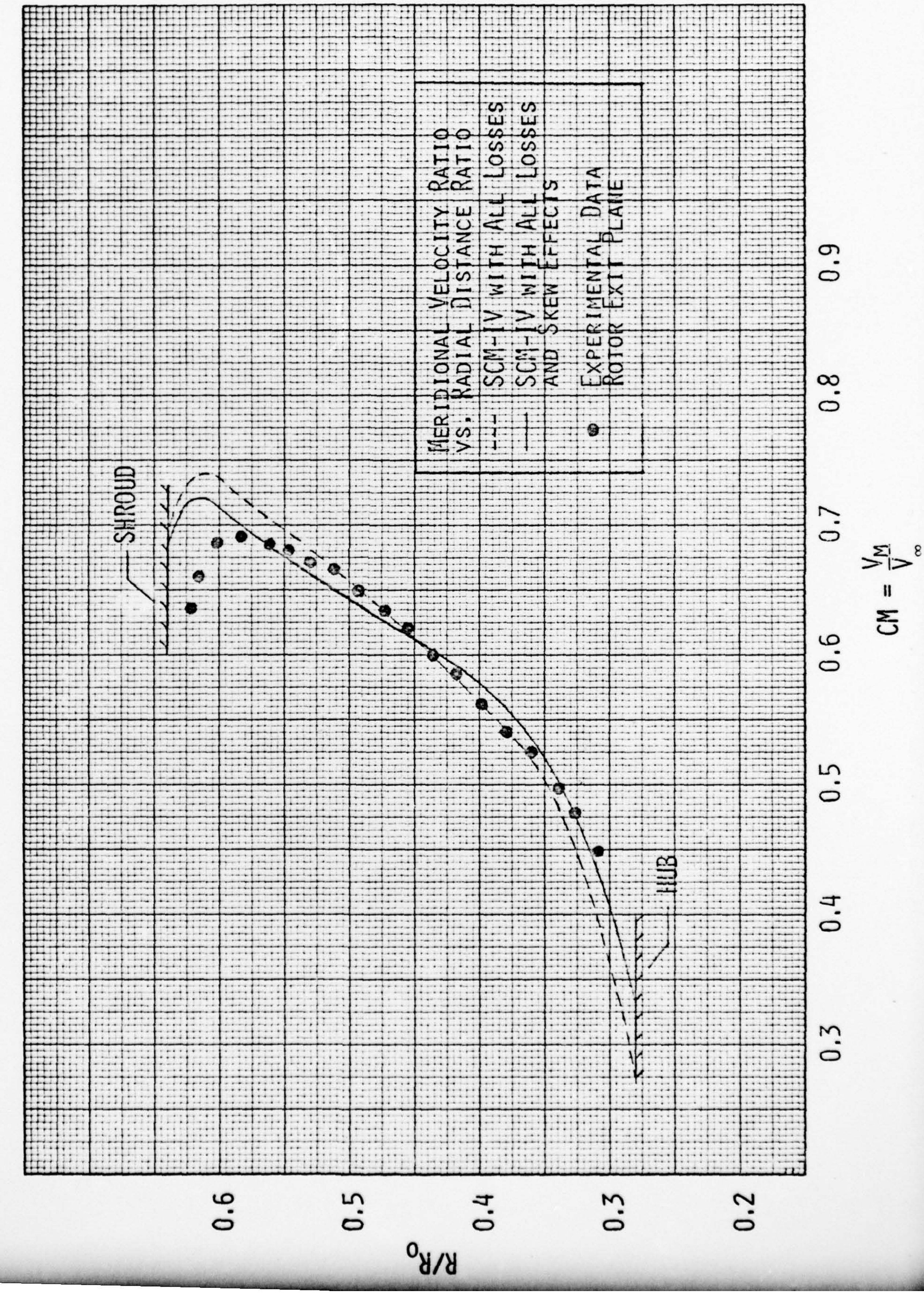
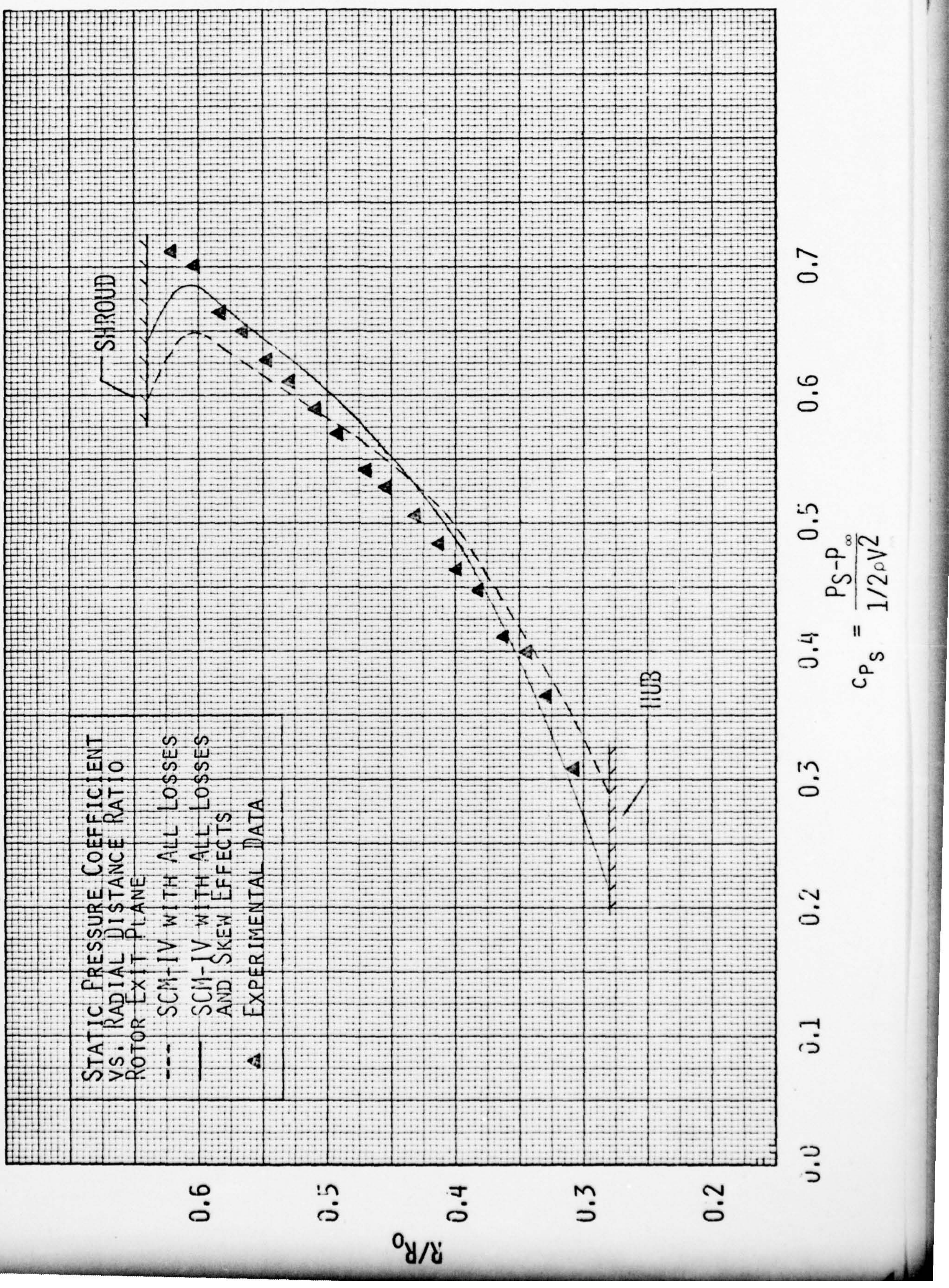


FIGURE 6. Comparison of Experimental and Calculated Data: Static Pressure Coefficient



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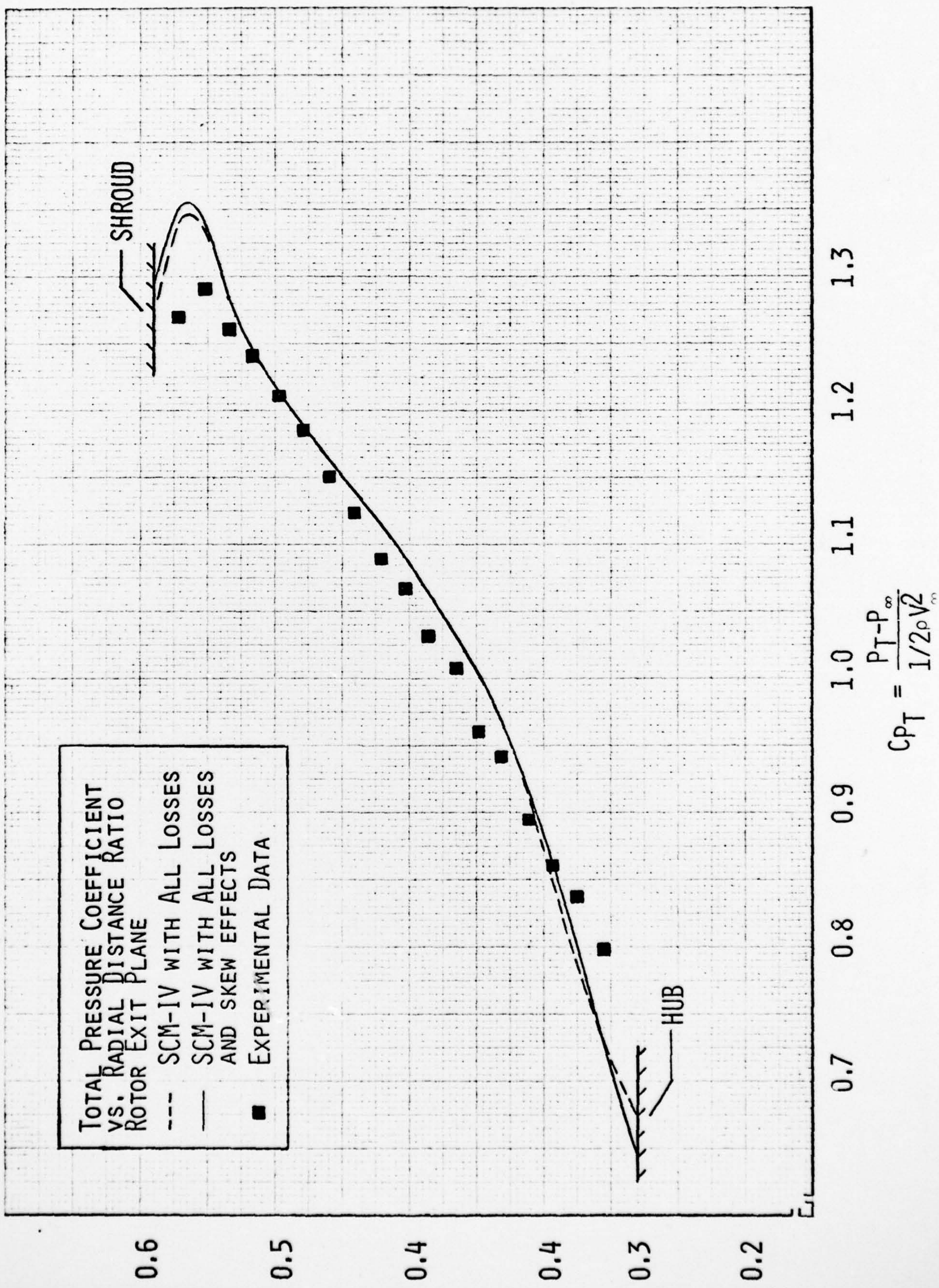

FIGURE 7. Comparison of Experimental and Calculated Data: Total Pressure Coefficient

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